Using Representations in Brownie Problems: What Do You Call a Third of a Half?

After working on an activity in which a given number of paper "brownies" are shared among a given number of people, for example, five brownies shared among three people, Ms. King looks over the students' work to determine what her next steps should be. She is particularly concerned with helping her students represent each situation and accurately name and record the fractional parts.

I saw that several students had shared the brownies correctly but didn't know how to accurately record the amounts. It was difficult for them to move between representing the situation and recording the quantity. A few did not have correct shares; some did not even have the required number of brownies in their work. I know that some students need to cut all the brownies up rather than sharing whole brownies first. Other students don't see that cutting up all the brownies is a possible strategy for solving the problems. Some students think that unless there is the same number of brownies as people, then it is not possible to share the brownies fairly.

My conversation with Yukio shows some typical difficulties. Even though earlier in the unit Yukio had successfully cut up one brownie to share with two, three, or more people and had also made the *Fraction Sets,* he was not able to make sense of the current situation. When I joined him, he was working on

the problem "five brownies shared among three people." He knew people were cutting up the brownies, but he wasn't seeing any connection between the number of people and the size of the pieces. He randomly cut pieces and glued them on the paper. He wasn't able to explain where the individual shares were on his paper nor was he able to explain if he had the total number of brownies in the original problem. He wasn't able to recombine the pieces to see if there really were five brownies. He had no sense of the sizes of the pieces. Like some other students, he started cutting; when he was done, he had no idea how big the pieces were or whether his solution worked.

At first he wanted to cut the brownies up in fourths. Since he had already tried unsuccessfully to solve the problem that way, I decided to redirect him. I thought that it would be useful for him to think about sharing whole brownies first. He would have fewer small pieces to keep track of, and it would be easier for him to be able to count up and find the five original brownies in his solution.

Teacher: So you have five brownies. What can you do to share them with three people?

He began to fold them in fourths.

Teacher: Do you need to cut them up to share them with three people?
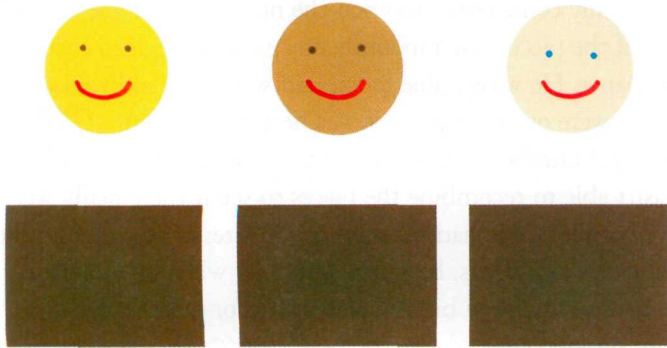
Yukio: You need more brownies.

I think he meant that he needed more brownies because there weren't enough *whole* brownies to share equally for everyone.

Teacher: You have five brownies and three people. Can you give anybody some brownie right now?

Yukio: Yes, I could give them each a whole.

Teacher: Yes. Why don't you do that?

Yukio gave one brownie to each imaginary person.

**Yukio:** But now we have two brownies left.

Teacher: Okay, is there something we could do with the two brownies that would share them fairly?

Yukio: Cut them in half?

I had hoped he would say thirds, but it is hard for kids to think about thirds. It seems easy for them to share with two or four people, but when faced with three people, they still want to cut into fourths. While there's something easy about halving halves, the idea of thirds and sixths is trickier.

Teacher: Okay, why don't you try that?

Yukio folded one of the remaining brownies, cut it in half and gave each half to two of the people.
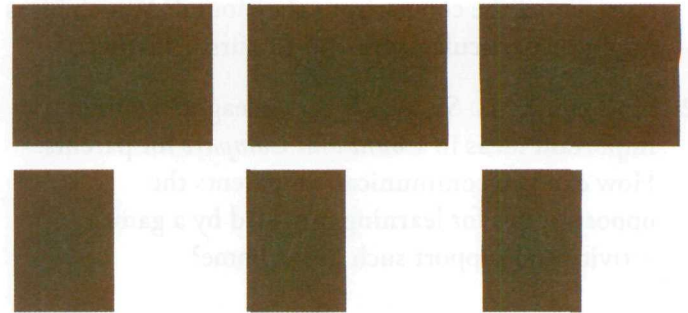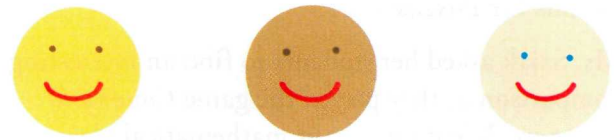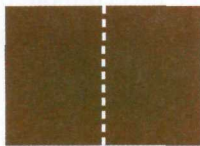
Then he picked up one of the whole brownies he'd already given out and was going to cut it in half.

Teacher: Let's leave these wholes here, Yukio.

I wanted him to leave those three whole brownies that he already distributed alone. I knew I was leading him, but I hoped this would help him develop some understanding about the relationships between how many people and how many brownies in these problems.

He picked up the remaining whole brownie, folded it, and cut it.
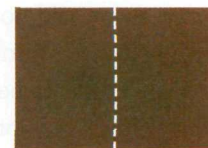
Teacher: What do you need to do to make it fair?

He gave one half to the third person.
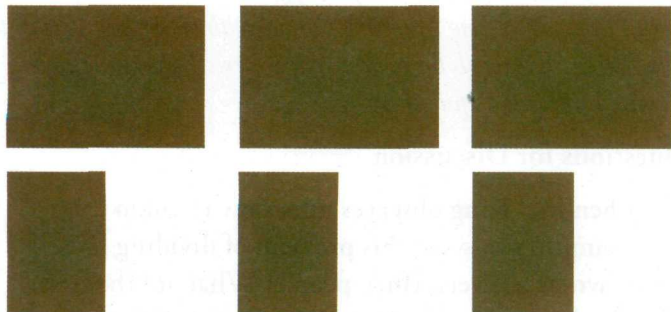
Teacher: Cool! Is it fair now?
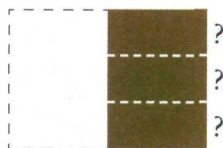
Yukio: Yes.

Teacher: How many brownies does each person have so far?

Yukio: One and a half.

Yukio looked at the half of a brownie he still had in his hand.

Teacher: But you still have a piece of brownie left? Don't cut it yet. (He starts to fold the half in half.) What do you need to do to that piece so that everyone has the same amount?

Yukio: Cut it into three pieces . . . into small thirds.

I was impressed that he was referring to the new pieces as thirds. A few minutes ago thirds did not appear to be a part of this thinking.

Teacher: A small third. Cool! You're right, but it's not a third.

Yukio: It's a fourth!

Teacher: It's not a fourth. But it has a name.

Yukio: Eighth!

Teacher: It's not an eighth either. Don't cut it yet.

I was afraid we'd never figure out what the piece was called if it was cut up.

Teacher: What makes it feel like it's a small third?

Yukio: Because there are three of these.

He has folded the half in thirds.

Teacher: Yes, there are three pieces. Don't cut it yet.

Yukio: So they're thirds, but they're small.

Teacher: Yes, they are small, and there are three of them. But are they a third of a brownie? What is it a third of?

Yukio: It's a third of a half.

Teacher: Yes, it's a third of a half. Don't cut it yet. Before we cut it up, we have to figure out how big these pieces are. You are right! They are a third of a half. But what is their name?

I reached for a whole brownie.

Teacher: Here's a whole brownie. (I folded it in half and then into thirds, just as Yukio had. I put it down next to the half he had folded into "small thirds.")
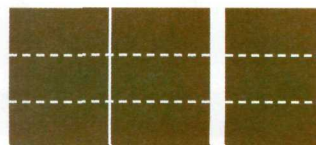
Yukio: It's a sixth!

Teacher: Why?

Yukio: Because there are three on this side and three on this side.

Teacher: Excellent! So when you cut up that half of your brownie, the pieces aren't going to be thirds, they're going to be sixths because they are a third of a half.

I thought it was important to restate Yukio's thinking. I hope it helps!

Teacher: I liked how you called them "small thirds," but they aren't thirds.

Yukio: They are sixths. There are six of them.

He points to the whole brownies we had folded.

I asked Yukio to glue the shares for each person on his paper and to write the number of brownies that each person got. When I returned later, I was pleased with the result.

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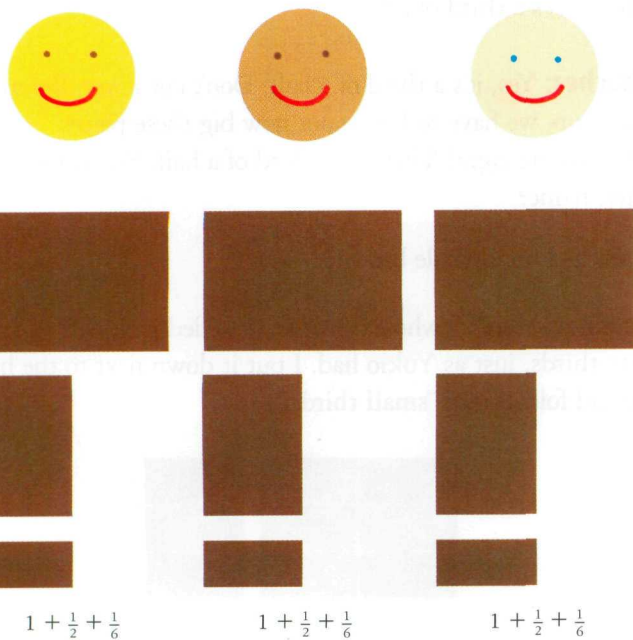
While many of Ms. King's students are comfortable dividing one brownie into fractional parts, representing and recording the fractions involved with dividing five brownies among three people proves to be more challenging. Because students are more experienced working with halves and fourths, dividing brownies among three people adds another layer of complexity for the students to represent. In this case, we see how Ms. King worked with one of her students to help him work through his confusion and correctly represent and identify each person's fair share.

Questions for Discussion

1. When Ms. King observes him, how is Yukio attempting to solve this problem of dividing five brownies between three people? What are the first steps Ms. King takes to help him solve and represent this problem?

2. What issues arise for Yukio as he tries to decide how to divide the remaining half brownie? What steps does Ms. King take to help Yukio work through the confusion he is having about what to call a third of a half?

3. Have you encountered similar confusion about sharing many brownies among the students in your classroom? What next steps would you put in place for Yukio or for students like him?